

## Chapter 2- Integers and Rounding Notes and Problems

### 2.1 Integers

Integers are mostly used in scientific work than in business. Out of superstition, commerce avoided the use of negative numbers for a long time. This is because people believed that only the gods of destruction could conceive of negative numbers. The signs “-” and “+” to the left side of counting numbers signify negative numbers and positive numbers, respectively. For example -4 is negative of counting number 4 and +4 is positive of counting number 4. These are shortened as negative 4 and positive 4. The relationship between negative and positive numbers is that, the sum of the negative and positive of the same counting number is zero. For example, the sum of -4 and +4 = 0. The negative and the positive of the same number are additive inverses of each other. The ancient Chinese used to colour negative numbers red and positive numbers black. This may be the source of account records written in red for loss and black for gain.

**Integers** are the set of positive whole numbers, negative whole numbers and Zero; ..., -3, -2, -1, 0, + 1, +2, +3, .... Below is a table that gives an idea of what positive and negative numbers represent in accounting.

<b>Positive (+ )</b>	<b>Negative ( - )</b>
Deposit	Withdrawal
Stock receipt	Stock sale
Credit	Debit
Replenishment	Depletion
Transfer in	Transfer out
Loan Payment	Loan

Indeed, in financial and managerial accounting the set of integers may express profit and loss or income and expenses. Think of negative integers as representing losses and expenses and positive numbers as profit or income. In terms of cash flow

statement, any item that potentially reduces cash is negative and any one that increases cash is positive. Integer as a mathematical concept is also used in stock exchange records and for analyzing budget statements. In stock exchange records, the prices of stocks at the close of the previous day are compared to prices of the preceding day. Any change is indicated with either positive or negative sign.

Similarly, in analyzing budget statements, the budgeted figures are compared with the actual money spent or received to indicate differences. Where budgeted figures are greater than the actual figures, the difference is indicated with a positive sign. On the other hand, if the actual figures are greater than the budgeted figures, any differences are indicated with a negative sign. In fact, where the total budgeted revenue exceeds the actual expenses the budget is in a surplus or positive position. And the opposite is described as deficit or negative. Furthermore, cash flow statement, which shows cash flowing in and out of a business, follows the same procedure. Any financial event that reduces the cash resources is a cash outflow and is indicated with a negative sign, where as events that increase cash are cash inflow and are marked with positive signs.

## 2.2 Arithmetic of Integers

Our purpose here is to illustrate how operations with integers work rather than to prove them. Since students might have learned these operations at other levels of education, this section serves as a review.

1. Multiplication or Division of *two negative* numbers, results in a *positive* number as the answer.

Example,  $-4 \times -3 = 12$ ;  $-14 \div -7 = 2$

**Example 1** Suppose \$950.00 is being deducted every two weeks to pay for your store mortgage. After two months,  $4 \times -950$  or  $-\$3800$  would have been taken out for mortgage. Now suppose the landlord decided to become a partner and remove the mortgage payment. We represent the “remove” by a negative number and find the answer by multiplying:

$(-4) \times (-950) = \$3,800$ . This is a concrete illustration of a negative times a negative resulting in a positive. (Note: Finding examples of business situations where negative  $\times$  negative equals positive are not easy to find.)

2. Multiplication or Division involving a ***negative and positive number***, results in a ***negative*** number as the answer.

Example:  $-4 \times 3 = -12$ ,  $4 \times -3 = -12$ ;  $14 \div -7 = -2$ ,  $-14 \div 7 = -2$

Take Notice:  $-4 \times 3 = 4 \times -3$ ;  $14 \div -7 = -14 \div 7$

**Example 2:** Suppose you used your credit card to buy 6 Christmas cards each costing \$5. You owe the credit card company,  $6 \times -\$5 = -\$30$

3. Multiplication or Division of ***two positive*** numbers, results in a ***positive*** number as the answer.

Example:  $4 \times 3 = 12$ ;  $14 \div 7 = 2$

### Example 3

Suppose you work as a sales person for a car renting company. A customer made three payments of \$233 each (note: for your company these are receipts). The total receipts =  $3 (\$233) = \$699$ . If another customer is behind her account for two months in the amount of \$142 each, use integers to describe this situation.

### Example 4

Progressive Express Services had the following cash flow items. Depreciation amount of \$3,500; loan interest payment of \$2,800; payment of supplies \$25,000; cash sales \$45,000; credit sales \$1,900; salaries and wages payments \$12,000; payment of account receivables \$8,200. Use integer to describe the effects of these events on the company's cash position.

### Solution

Depreciation	+ 3,500 (addition to cash)
Loan payment	- 2,800 (deduction from cash)
Payment of supplies	- 25,000 (deduction from cash)
Cash sales	+ 45,000 (addition to cash)

Credit sales	0 (has no effect on cash, so it is 0)
Salaries & wages	-12,000 (decreases cash)
Payment on account receivable	+8,200 (addition to cash)

Note that depreciation is the spreading of the cost of an asset such as machinery or furniture over its useful life. It only reduces cash at the time it was purchased. After that, charging a bit of that cost to a company's financial statement has no effect on the cash resources of the company. It is only a prudent accounting procedure with no consequences on a company's cash resources. In other words, depreciation is an accounting procedure with no effect on cash. The same idea applies to any losses that a company may write off periodically.

### 2.3 Integers Problems (Answers follow)

1. Simplify each of the following.

- a)  $(1 - 2) - (6 + 3)$     b)  $(17 - \{14 - (21 - 31)\})$     c)  $-49 - (-68)$   
 d)  $-52 - (-96)$     e)  $(2 - 9) - (2)$

2. Use integers to describe these statements

- a) An income of \$1500 followed by an income of \$1500  
 b) An income of \$1000 followed by expenses of \$400  
 c) A profit of \$2000 followed by a loss of \$1250  
 d) A loss of \$40 followed by a loss of \$650  
 e) Expenses of \$1300 followed by earnings of \$2500  
 f) A loss of \$150 followed by a profit of \$140

3. John's cell phone bill for every month is \$31.50. For three months now, he has not been able to make any payments and Cell Phone Company is threatening to assign the debt to a collection agency. Use your knowledge of integers to indicate how much John owes to the Cell Phone Company.
4. Sam has a chequing account. At the first day of the month, he deposited \$500 to a previous balance of \$221. He later wrote six cheques for \$135, \$250, \$25, \$60, \$140, and \$75. He made another deposit of \$300. If the bank service charge was \$10, what was Sam's account balance at the end of that month?
5. From the following list, select pairs that are additive inverses.  
a) -2, 2   b) -3, -3   c) 7, -8   d) 0, 0   e) 2, 2   f) 1 -1
5. From the following list of words, pick a pair of words that explain additive inverses. Top, forward, bottom, positive, backward, negative, increase, decrease, below, up, above, down, south, right, north, left, add, multiply, subtract, divide.
6. Insert a pair of brackets to make the following a true statement.  
 $1 - 2 + 3 - 4 + 5 - 6 = -1$
7. Classify each of the following true or false.  
a) Every integer is a whole number.  
b) Every counting number is an integer.  
c) Every integer has an additive inverse.  
d) The set of integers is the same as the set of additive inverses of whole numbers.  
e) Zero is the only integer whose opposite or inverse is zero.
8. State under what condition will the product of two factors be  
a) greater than 0?   b) equal to 0?   c) less than 0?

9. Multiply each.

a)  $(-16) \times (+12)$     b)  $(+3) \times (+24)$     c)  $(-24) \times (-5)$     d)  $(-5) \times (-8)$

10. Arrange each list of integers from least to greatest.

a) 0, +3, -1, +8, -7, -5, +1

b) +6, -3, 0, +3, +10, -10

c) -2, -5, -8, -14, 0, +9, +2

d) 0, -47, +32, -7, -6, +8, -20

11. Simplify each of the following.

a)  $(-7)(-2) + (-3)(-4)$

b)  $(-3)(+2) - (+3)(+2)$

c)  $(-3)(-7) - (+2)(+4)$

12. Find: a)  $+26 \div -2$     b)  $\frac{-125}{-5}$     c)  $-42 \div -14$     d)  $0 \div -5$

13. Find each of the following:

a)  $\frac{-22}{+2}$     b)  $\frac{-24}{-12}$     c)  $\frac{+52}{-4}$     d)  $\frac{+18}{+6}$     e)  $\frac{-35}{+1}$     f)  $\frac{0}{18}$

14. Complete the following statements.

a) All positive numbers are..... than zero.

b) All negative numbers are..... than zero

c) Zero is always..... than a negative number.

d) Zero is never ..... than a positive number.

e) Positive numbers are.... than negative numbers.

f)  $-6$  is ..... than  $+6$ .

15. A hockey player was given a rating of 10 when he joined his team.

While playing his shift, the coach rated his line positive, negative, or zero depending on whether they scored, gave up a good goal or held the other team on each shift. For one game the rating he received each shift was 0, 0 -1, +1, +1, 0, -1 and -1.

What is his rating after the end of the game?

16. The temperature in January 20, 2006 at 07:00 was  $-14$  C. How many degrees must the temperature rise before it is  $+12$  C?

17. In each of the following cases, represent the event by a difference of integers and state the result.

- You have \$30 and lose a \$10 bill.
- In 1 week Joe earns \$400 and spends \$250
- Pete has \$1000 in the bank and writes a cheque for \$360
- The temperature is  $-5^{\circ}\text{C}$  and it drops by  $15^{\circ}\text{C}$ .

18. True or false.

- In a division problem the dividend should never be zero.
- The only way a quotient can be zero is for the dividend to be zero.
- It is impossible to divide a number by zero.
- Every time an integer is divided by itself, the quotient is  $+1$ .
- The integers  $-2$  and  $+2$  are inverses.
- Zero is the only integer that is an inverse of itself.
- If the product of two integers is zero, then at least one of the factors is zero.
- If the quotient of two integers is negative, then the dividend is not negative.
- If the product of two integers is negative and one of the integers is positive, then the other integer is negative.

19. Fill in the blank with an integer, so that each of the following is true.

- a)  $\underline{\quad} + 8 = -5$     b)  $\underline{\quad} - 7 = 6$     c)  $-5 + \underline{\quad} = 11$     d)  $-1 + (\underline{\quad})(-5) = 14$

20. Simplify completely.

- a)  $(-3) \times (4) \times (-1)$     b)  $(-2) \times (-1) \times (-3)$     c)  $(-3)(-2)(+4)(+3)$

21. Fill in the blanks.

- a)  $(\underline{\quad})(+2) = -18$     b)  $(-24) \div \underline{\quad} = +4$     c)  $(-100) \times \underline{\quad} = 400$     d)  $\underline{\quad} \div (-2) = 0$

22. Fill in the blanks.

- a) If  $+100$  means 100 kilometres north, then  $-100$  means 100 kilometres \_\_\_\_\_
- b) If  $+35$  means the temperature has increased 35 degrees, then  $-35$  means the temperature has \_\_\_\_\_ 35 degrees.

23. Complete each sentence.

- If  $x$  and  $y$  represent positive integers, then  $x + y$  represents a \_\_\_\_\_ integer.
- If  $x$  and  $y$  represent negatives, then  $x + y$  represents a \_\_\_\_\_ integer.

23. George can buy a stereo either for \$285.00 cash or for \$40.00 down and \$25 per month for a year. How much will he save by paying cash?

24. Write integers which are,  
 a) Less than -3    b) Greater than -4 but less than -3  
 c) Less than -3 but greater than -7
25. If  $x$  represents a negative integer and  $y$  positive integer, then what is  $x \div y$ ?  
 If  $y$  represents a negative integer and  $y$  positive integer, then what is  $x \times y$ ?
26. Find the value of each of the following.  
 a)  $A - B + C$     if  $A = 3, B = 3, C = -1$   
 b)  $-K - M - N$     if  $K = -4, M = 9, N = -1$
27. Evaluate. a)  $(28 \div 4) \times (-3)$     b)  $(-72) \div (-8)$     c)  $(-4) \times 9$     d)  $-5 \times (-12)$
28. What is the product of each? a)  $(-5) \times \{7 \times (-3)\}$     b)  $\{(-5) \times 7\} \times (-3)$   
 Compare your answers in (a) and (b). Do you think regrouping is allowed in multiplication of integers? Explain why.

## Chapter 2: Integers Answers to Review, Exercises, and Assignments

### Section 2.3 Exercises

1. a) -10    b) -7    c) 19    d) -148    e) -9
2. a)  $+1500 + \$1500$ ; b)  $+1000 - \$400$ ; c)  $+2000 - 1250$ ; d)  $-40 - 650$ ; e)  $-1300 + 2500$ ;  
 f)  $-150 + 140$
3.  $-\$94.50$     4.  $\$326$ ; 5.  $(-2, +2), (+1, -1), (-3, +3), (0, 0)$
5. Forward- backward; positive-negative; increase- decrease; top- bottom  
 Up – down; above-below; South-North; Right- left; Add- subtract; multiply-  
 divide.
6.  $(1-2+3) - (4 + 5 - 6) = -1$     7. a) True. b) True. c) True    d) True e) True.
8. a) Both factors are positive; b) Either one is zero c) One is positive and the other negative.
9. a) -192    b) +72    c) +120    d) +40.
10. a) -7, -5, -1, 0, +1, +3; b) -10, -3, 0, +3, +6, +10; c) -14, -8, -5, -2, 0, +2, +9  
 d) -47, -20, -7, -6, +8, +32.
11. a) +26; b) -12; c) +13.    12. a) -13, b) + 25, c) +3    d) 0
13. a) -11, b) 2, c) -13, d) 3, e) -35, f) 0.    32) 22 points
14. a) Greater, b) Loss, c) Greater, d) Greater,    e) Greater, f) Loss
15.  $10 - 1 = 9$
16. 17C    417 a) \$20, b) \$150, c) 640, d) -20
18. a) False    b) True    c) True    d) True    e) True    f) True    g) True    h) False    i) True.

19. a) -13, b) +13, c) +16 d) 3  
 20. a) +12, b) -6, c) +72. 21. a) -9, b) -6, c) -4, d) 0.  
 22. a) South, b) Dropped or fallen or decreased,.  
 23. a) Positive, b) Negative.  
 24. a) -4, -5, -6, ..., b) impossible c) -6, -5, -4  
 25. a) Negative, b) Negative.  
 26. a) -1 b) -4. 27. a) -21, b) 9, c) -36, d) 60  
 28. a) 105 b) 105, Yes because the results are the same.

## 2.4 Decimal Point and Decimal Numbers

Arab mathematicians added the Decimal point to the Hindu Number system to make it possible to express fractions in the same format as whole numbers, where the position of a digit determines its value. Initially the decimal point was a bar on the units position, positions to its right decreases by multiples of  $\frac{1}{10}$ , and positions to its left increases by multiples of 10.

An Arabic scholar represented a decimal number as  $248\bar{3}57$ , for the position of the digits are 3 units, 5 in the tenths or  $\frac{1}{10}$ , 7 in the hundredth or  $\frac{1}{100}$ , 8 in the tens or 10, 4 in the hundreds or 100, and 2 in the thousands or 1000. The number is now written 2483.57.

## 2.5 Round off and precision

A number for a quantity may be the exact number of the quantity or an approximation of the quantity. For certain quantities, only an approximate number can be given, for example, measuring length, or weight. For other quantities, only an approximate value makes sense, for example, the population of the world, Canada, or a city. In other situations, approximate numbers are used to make 'rough' estimates. For example, the total cost of items given the unit cost. Also in expressing fractions in decimal, the question of acceptable number of decimal places comes up. For example, the fraction  $\frac{1}{3}$ , as a decimal is .33333..., so what is the approximate value?

That type of approximation is called Round-Off. The round-off number may be greater or less than the exact number. It is therefore important that, from only the round-off number, the maximum possible value, and the minimum possible value, for the exact number can be found. From this, the maximum difference between the exact number and the round-off number can also be found. This is very important in practice, as it may cost or enrich a business in monetary transactions.

**Example 1:** If Maya gives her age as ten years, then by convention it means Maya could be a few seconds older than ten years or a few seconds younger than eleven years. What can be definitely said is that, Maya's exact age is more than ten years and less than eleven years. The maximum difference between her round-off age and exact age is one year in case she is seconds from eleven.

Numbers are rounded-off to a specified position, for example units, hundreds, hundredths position. It is similar to measuring a length by a ruler with the smallest calibration in millimeters, centimeters, meters, or kilometers. The measurement cannot be more accurate than to the nearest unit of smallest calibration and the deviation from the exact length, about half the unit. Round-off to a specified position is similar to measuring with a ruler calibrated with the position as its smallest unit. Example, round-off to the hundreds position is similar to measuring with a ruler, calibrated in units of 100. Deviation of the measurement from the exact length is half the unit, 50.

**Example2:** Round off the following numbers to the nearest unit or whole number.

- a) 437.499   b) 437.501   c) 438.385   d) 438.499   d) 438.5001

Solutions: a)  $437.499 = 437$  (nearest unit)

b)  $437.501 = 438$  (nearest unit)

c)  $438.385 = 438$  (nearest unit)

d)  $438.499 = 438$  (nearest unit)

e)  $438.501 = 439$  (nearest unit)





difference between the number obtained by round-off to the nearest ten and the exact number is 5, which is half of 10, and for round off to the nearest thousandth it is .0005, which is half of thousandth (.001).

An important application of round-off and maximum round-off error is in the interpretation of given numerical values. If Maya gives her age as ten years, it is understood that Maya's exact age is somewhere between ten years and under eleven years. What, if Maya gives her height as 1.24 m? Does she measure exactly 1.24 m or, is this the round-off value of her height? In practice, it is interpreted as her height to the hundredth of a metre, that is, the measurement of her height round-off to the nearest hundredth. This means Maya's exact height is between 1.235 m and 1.245 m; that is Maya is at least 1.235 m and at most just less than 1.245 m and maximum error is .005 m.

In practice, any given number is interpreted as the round-off number of an unknown exact number. If the given number is an Integer then it is interpreted as the result of an unknown number, round-off, to the position of the first non-zero digit from the right to left in the given number. Example, for 309200, 2 is the first non-zero digit from the right to left, and 2 is in the hundreds position, so an unknown number round-off to the nearest hundred is 309200. The maximum difference between this unknown number and 309200 is half of hundred, which is 50. This unknown number is between 309150 and 309250. If the given number has a decimal part, then it is interpreted as the result of an unknown number, round-off to the position of the last decimal place (whether the last digit is zero or non-zero) of the given number. Example, for 23.037, the position of the last decimal place is thousandth, so an unknown number round-off to the nearest thousandth (3 decimal places) is 23.037. The maximum difference between 23.037 and the unknown number is half of a thousandth (.001), which is .0005. The unknown number is between 23.0365 and 23.0375. For 5.30, the maximum difference between the unknown number and 5.30 is half of a hundredth (.01), which is .005. The unknown number is between 5.295 and

5.305. This is because the unknown number round-off, to the hundredth (two decimal places) is 5.30.

### **2.5 Arithmetic and Round-off**

Rounding introduces ‘error’. To minimize errors from rounding off in calculations as much as possible, the rounding is applied only to the final calculated value. Intermediary values, if rounded off, must be rounded off to a position which is less than at least twice the rounded off position of the final answer.

Example:

1. Multiply 2.236 and 5.642 and round-off your answer to 2 decimal places.

$$2.236 \times 5.642 = 12.615512, \quad 12.615512 = 12.62 \text{ (2 decimal places)}$$

On the other hand  $2.236 = 2.24$  (2 decimal places),  $5.642 = 5.64$  (2 decimal places) and  $2.24 \times 5.64 = 12.6336$ ,  $12.6336 = 12.63$  (2 decimal places)

Note that 12.63 is further away from 12.615512 than 12.62 is from 12.615512. So the error from using 12.63 is bigger than the error from using 12.62 as the product of the two original numbers.

2. Add 2.436 and 5.352 and round-off the answer to the nearest unit or whole number.

$2.436 + 5.352 = 7.788$ ,  $7.788 = 8$  (to the nearest unit or whole number). On the other hand,  $2.436 = 2$  (to the nearest unit),  $5.352 = 5$  (to the nearest unit) and  $2 + 5 = 7$ . Note that 7 is further from 7.788 than 8 is from 7.788. So, the error from using 7 is bigger than the error from using 8 as the sum of the two original numbers.

### **2.6 Precision and Round-Off**

Precision is a relative term or concept. A distance can be measured to the nearest metre by using a measuring tape calibrated in metres, or can be measured to the nearest kilometer by using a measuring tape calibrated in kilometers. The measurement to the nearest metre is closer to the exact measurement of the distance than the measurement to the nearest kilometer. So measurement to the nearest metre is more precise than measurement to the nearest kilometer. In the same numerical

value of a quantity, round-off or given to the nearest tens is more precise than if given to the nearest hundreds.

A numerical value of a quantity is more precise the lower the round-off position. A measurement to 4 decimal places is more precise than a measurement to 3 decimal places. If actual sales were \$4875.50, a salesperson who quotes sales for the day as being \$5 000 is being less precise than the one who quotes the same sales as \$4875.

## 2.7 Review, Exercises and Assignments

1. Write each number to the stated decimal places to complete the table.

Number	2 Decimal Places	1 Decimal place
2.459		
10.726		
0.003		
1.999		
0.609		
05.446		
0.606		
0.084		
4.909		
20.009		

2. Round off each of the following to the nearest 10.

146	18.97	215
147	24.61	345
946.3	48.67	537

3. Round off each of the following to the nearest 1000

- a) 2002   b) 2307   c) 870.99   d) 999   e) 3121   f) 1709

4. Round off each of the following number to two decimal places.

- a) 3.1671   b) 8.134   c) 5.315   d) 7.245   e) 7.1451   f) 3.1416

5. Round off the numbers below to the indicated places.

- a) 2453; tens  
b) 196; tens  
c) 1431; hundreds  
d) 2576; hundreds  
e) 18227; thousands  
f) 10,760; hundreds  
g) 102,876; thousands

6. Which is the smallest number in each case?

- a) 0.095, 0.1, 0.16   b) 3, 0.333, 0.3   c) 0.85, 0.9, 0.099   d) 0.245, 0.268

7. Round to the nearest tenth, hundredth, and whole number.

- a) 45.943   b) 0.845   c) 3.899   d) 0.08   e) 0.967

8. The number 17.49 is closest to which number? a) 18   b) 17   c) 17.4   d) 17.5

9. Divide each. Round off your answer to the hundredths.

- a)  $1.65 \div 11$    b)  $0.105 \div 15$    c)  $54.68 \div 25$    d)  $365.04 \div 27$    e)  $291.48 \div 34$

10.  $0.4 \times 480$  is the same as:

- a)  $480 \div 2$    b)  $48 \times 50$    c)  $0.40 \times 480$    d)  $480 \div 5$

**Chapter 2: Section 2. 7- Answers to Exercises**

1. #	2 decimal places	1 decimal place
2.459	2.46	2.5
10.726	10.73	10.7
0.003	0.00	0.0
1.999	2.00	2.0
0.609	0.61	0.6
05.446	5.45	5.5
0.606	0.61	0.6
0.084	0.08	0.1
4.909	4.91	4.9
20.009	20.01	20.0

2. 150, 19, 220

150, 25, 350

950, 50 540

3.a) 2000 b) 2000 c) 1000 d) 1000 e) 3000 f). 2000

4. a) 3.17 b) 8.13 c) 5.32 d) 7.25 e) 7.15 f) 3.14

5. a) 2450, b) 200, c) 1400, d) 2600, e) 1800, f) 10800 g) 103,000

6. a) 0.095 b) 0.3, c) 0.099 d) 0.245.

7. a) 45.9, 45.94, 46. b) 0.8, 0.85, 1. c) 3.9, 3.90, 4. d) 0.1, 0.08, 0

8. d.. 9. a) 0.15 b) 0.01 c) 2.19 d) 13.52 e) 8.57

10. c.