

4 Percent Notes

4.1 Percent is a special ‘fraction’ with denominator of 100. The numerator of a percent can be a whole number or a fraction but the numerator of a fraction is always a whole number.

Percent symbol %, is the fraction, $\frac{1}{100}$, that is, $\% = \frac{1}{100}$.

Example:

1. 50% is also the fraction $\frac{50}{100}$
2. 127% is also the fraction $\frac{127}{100}$
3. 12.5% is also the fraction $\frac{12.5}{100} = \frac{125}{1000}$
4. $33\frac{1}{3}\%$ is also the fraction $\frac{33\frac{1}{3}}{100} = \frac{100}{300}$

Similarly, a given fraction has an equivalent percent, which is a ‘fraction’ with denominator 100.

Example:

1. $\frac{3}{4} = \frac{3}{4} \times \frac{100}{100} = \frac{\frac{3}{4} \times 100}{100} = \frac{75}{100} = 75\%$ So $\frac{3}{4}$ as a percentage is 75%.
2. $\frac{5}{7} = \frac{\frac{5}{7} \times 100}{100} = \frac{\frac{500}{7}}{100} = \frac{71\frac{3}{7}}{100} = 71\frac{3}{7}\%$ So $\frac{5}{7}$ as a percentage is $71\frac{3}{7}\%$.
3. $1.32 = \frac{1.32 \times 100}{100} = \frac{132}{100} = 132\%$ So 1.32 as a percentage is 132%.
4. $5 = (5 \times 100)\% = 500\%$ So 5 as a percentage is 500%.

Any number C (whether is a fraction, decimal, or integer) as a percentage is $(C \times 100)\%$.

4.2 Decimal Form of Percent

Percent can also be written in decimal form by dividing it by 100, which is the same as placing a decimal point two digits to the left.

Example 3

$$5\% = 5 \div 100 = 0.05$$

$$14\% = 14 \div 100 = 0.14$$

$$20\% = 20 \div 100 = 0.20$$

$$35.2\% = 35.2 \div 100 = 0.352$$

$$\frac{1}{2}\% = 0.50 \div 100 = 0.005$$

$$33\frac{1}{3}\% = 33.33 \div 100 = 0.3333$$

4.3 Ratio form of Percent

Percent can also be written in ratio form with the percent being the first term and 100 the second term.

Example 4 $10\% = 10:100$; $54.5\% = 54.5:100$; $15\% = 15:100$; $75\% = 75:100$

4.4 Calculating Percentages

Example 1

In 2007 in Canada, John bought a sofa chair for \$299. He had to pay the GST of 6% and PST of 8% on his purchase. a) How much was the GST? B) How much was the PST? c) Will it make any difference if you were to apply a single rate of 14% sales tax? Note: GST in 2007 was still 6% and the HST had not yet been used.

Solution

a) $\text{GST} = 299 \times 0.06 = \17.94

b) $\text{PST} = 299 \times 0.08 = \23.92

c) No, it will not make any difference since $0.14 \times 299 = \$41.86$. This is the same as $\$17.94 + 23.92$.

Example 2

Mary's annual salary is \$54,000. Her raise for this year is 3.5%. How much raise did she get?

Solution

Raise = $54000 \times 0.035 = \$1890$. She got a raise of \$1890.

Example 3

An airplane was 85% full when it had 425 passengers. What is the full seating capacity of the airplane?

Solution

It should occur to you immediately you read the question that the airplane is full when it is 100%. This implies that a full-seating capacity occurs when it is 100% full

Let y represent the full seating capacity of the airplane. Then

$$85\% \text{ of } y = 425$$

$$\frac{0.85y}{0.85} = \frac{425}{0.85} \quad \square \text{ Divide each side by } 0.85$$

$$y = \frac{425}{0.85} = 500.$$

Alternatively, we could proceed to solve the problem as follows:

$$85\% \text{ full} = 425$$

$$1\% \text{ full} = 425 \div 85 = 5$$

$$100\% \text{ full} = 5 \times 100 = 500$$

The full seating capacity of the plane is 500.

The plane has a full-seating capacity of 500.

4.5 Percent increase and decrease

a) Finding Percent Rate of Increase or Decrease

Percent is often used to show either increase or decrease in amount. The price of lumber, for example, could increase from \$2.40 a metre to \$2.50. Hydro rate may jump from \$0.0530 per kilowatt hours to \$0.0570 per kilowatt hours. A paper manufacturing business may reduce the prices of its computer printing paper products from, say \$3.97 per 500 sheets to \$3.27. The increase or decrease in the amount can be conveniently expressed in terms of percentages. The following procedures are used.

$$\text{Rate of change (Increase)} = \frac{(NA - OA) \times 100}{OA}, \text{ where } NA = \text{new amount and } OA$$

=Original amount.

Where NA is the new amount, and OA is the original amount.

$$\text{Rate of change (Decrease)} = \frac{(OA - NA) \times 100}{OA}$$

Example 1

Mary's hourly wage was increased from \$14/h to \$15.50/h. What is the percentage?

Increase in her wage rate?

Solution 1

The original amount is \$14 and the new amount is \$15.50. Therefore, this is an increase.

$$\text{Change} = \frac{(15.50 - 14) \times 100}{14} = 10.7 \Rightarrow \text{Mary's hourly rate increased by about 11\%}.$$

Example 2

In a survey of 400 people, it was found that 240 watched *Everybody Loves Raymond*. What percent does not watch that show?

Solution 1

The original amount is 400. And those who watch that show are 240.

$$\text{Change} = (400 - 240) \div 400 \times 100 = (160 \div 400) \times 100 = 40\%$$

40% of the people surveyed do not watch that show.

b) Finding the rate of Percent Change

Instead of finding the percentage change, sometimes we have a situation where we are given the percent and our task is to find the new quantity. In this case, we have two procedure options, both of which will give us the same ultimate answers. However, the procedure we should use depends on the situation.

1) We may calculate the percentage increase or decrease and add or subtract it from the original quantity. We have, Original Amount + increase, or Original Amount – decrease

2) We may calculate the new amount straight away by using the given percent.

Now let us have an illustration of both procedure options.

Example 1

What is the number when \$40 is increased by 25%?

Option 1 (Calculating the percent increase or decrease separately)

The original number is \$40 and the change (increase) required is 25% of \$40.

$$40 + (0.25 \times 40) = ?$$

$40 + 10 = 50$. The new amount is \$50.

Option 2 (Calculating the new amount)

The original quantity is \$40 and the change (increase) is 25% of \$40.

The quantity \$40 represents 100% and must be increased by 25%. So the new quantity must be 125% (100 + 25).

Thus, 40: 100

N: 125

By cross-product, $100N = 40 \times 125$

$$100N = 5000 \text{ (Divide each side by 100)}$$

$$N = 50 \text{ (or } 40 \times 1.25\text{)}$$

The new amount is 50.

Example 2

What is the amount due when a debt of \$350 is reduced by 20%?

Option 1

The original amount is \$350 and it is to be reduced by 20%.

$$350 - (20\% \text{ of } 350) = ?$$

$$350 - (0.20 \times 350) = ?$$

$$350 - 70 = \$280$$

Option 2

Since the original amount of \$350 is to be reduced by 20%, 80% (100% - 20%) of the amount will be left. Therefore, $0.80 \times 350 = \$280$ is the new amount.

Example 3

After the price of a TV set was reduced by 15%, the sale price paid was \$245.

a) What was the price of the TV set before the reduction? b) What was the amount of reduction?

Solution

a) We can use any of the procedural option to answer the questions. Note that we are required to find the original price of the TV.

Thus, $100\% - 15\% = 85\%$. Since the original amount was multiplied by 85% to get the new amount of \$245, we should divide the new amount by 85% in order to get the original amount. $245 \div 0.85 = 288.24$ is the original amount.

b) To find the amount of the reduction we may use one of these. $288.24 \times 0.15 = \$43.24$, or $288.24 - 245 = \$43.24$.

4.6 Commercial Discounts and Mark up

Example 1

A portable CD player costing \$199.00 is on sale for \$99. (a) How much is the discount? (b) What is the rate of discount?

Solution

a) List Price – Discount = sale price
 $199 - 99 = 100$. The discount is \$100

b) Discount rate = $\frac{100}{199} \times 100 = 50.25\%$

Example 2

Ahmed wants to buy a new carpet for his house. The new carpet costs \$200. One day he saw the carpet being offered for 25% off the purchase price. (a) How much money does he save by buying the carpet? b) What is the net price?

Solution

a) Amount of discount = Rate of discount \times List price
 $= 0.25 \times 200 = \$50$, Ahmed saves \$50.00

b) Net price = List price – Discount amount
 $= 200 - 50 = \$150$

Instead of calculating the amount of discount and then deducting it from the list, the net price or net cost can be calculated by using the more efficient formula developed in the illustration below.

In example 2, since the discount is given as a percent of the list price, the following is obvious:

List.....\$200 – 100% of List price
 Less discount.....\$50 – 25% of List price
 Net price or Net cost 150 – 75% of List price

Note that the 75% is called the net cost factor or net factor. It is abbreviated as NCF and obtained by deducting 75% from 100%. That is, $NCF = 100\% - 75\%$. We should let d represent discount and L be the List price. In general, this could be written as, $NCF = (1.00 - d \%) \times L$ or

$NCF = (1 - d) L$, omitting the decimal point and the multiplication sign.

Using this efficient procedure, the Net price in example 2 could be calculated as follows. $NCF = (1 - 0.25)200 = 0.75 \times 200 = \150

In the case of series of discounts, that is more than one discount, the formula becomes, $NCF = (1 - d_1)(1 - d_2)(1 - d_3 \dots (1 - d_n)L$. In this formula, d_1 is the first discount, d_2 is the second discount, d_3 is the third discount and so on.

Example 3

More Store Corner is having a sale event. Ladies' jeans pants regularly priced at \$69 is being sold for 20 off and additional 10% on Saturdays only. Felicia bought 2 jeans on Saturday. How much does he pay for them?

Solution

$$\begin{aligned} NCF &= (1 - 0.20) (1 - 0.10) 69 \\ &= (0.80)(0.90)69 \\ &= \$49.68 \end{aligned}$$

In the above example, we can use rewrite the formula to calculate a single equivalent rate of discount, instead of applying series of discount to the base quantity. To rewrite the formula, we have to drop the L, so that we have,

$$\text{SDR} = \{1 - (1 - d_1)(1 - d_2)(1 - d_3)\dots(1 - d_n)\}.$$

Let's use the above formula to test the accuracy of the answer in example 3.

$$\begin{aligned}\text{SDR} &= \{1 - (1 - 0.20)(1 - 0.10)\} \\ &= \{1 - (0.80)(0.90)\} \\ &= (1 - 0.72) \\ &= 0.28\end{aligned}$$

The single rate is 28%. If we apply it to \$69, we get \$19.32 which is the discount amount. To get the net price as follows: $\text{NP} = 69 - 19.32 = \49.68 .

The most important thing about this formula is that you should be able to convert percents into decimals.

ii) Mark up

Businesses are set up for the primary purpose of making profit. So, businesses always add up their expenses, called overhead, and profit margin to the cost price of the goods or services they sell or produce in order to arrive at the selling price. The profit margin may be expressed as percent of the cost price of the goods/service or the selling price. The following relationship is therefore true,

$$\text{Selling price} = \text{Cost of buying} + \text{Expenses} + \text{Profit}$$

Using letters, it becomes, $\text{SP} = \text{CP} + \text{E} + \text{P}$, where CP is the cost of buying the product or producing it, SP is the selling price, E is the expenses, and P is the profit.

Example 1

A retailer has some skirts that cost her \$48 each. Her overhead expense for each skirt is \$3. She wants to sell them at a profit of 15% of the cost. What price should she sell each skirt?

Solution

$$\text{SP} = \text{CP} + \text{E} + \text{P}$$

$$SP = 48 + 3 + 15\% \text{ of } 48$$

$$SP = 48 + 3 + 0.15 \times 48$$

$$SP = 48 + 3 + 7.20$$

SP = \$58.20. She should sell each skirt at \$58.20 in order to realize 15% profit on cost.

Example 2

A pair of shoes cost a retailer \$32 and he sells each for \$44.80, including overhead expenses. What is his rate of profit based on the cost?

Solution

$$SP = \$44.80, CP = \$32,$$

$$\text{Rate of Profit on cost} = \frac{(SP - CP) \times 100}{CP}$$

$$= \frac{(44.80 - 32) \times 100}{32}$$

$$= 40\% \text{ (approximately)}$$

The retailer's rate of profit is about 40% on cost.

Example 3

A head of lettuce costs a retailer \$0.45. (a) At what price should it be sold to make a profit of 40% on the selling price? (b) What is the actual profit on each lettuce head?

Solution

a) Cost price = \$0.45, SP=? P= 40% on selling price

$$SP = CP + E + P$$

$$SP = 0.45 + 0 + 0.40(SP)$$

$$SP = 0.45 + 0.40SP$$

$$SP - 0.40SP = 0.45$$

$$0.60SP = 0.45 \text{ (divide both sides by 0.60)}$$

$$SP = \frac{0.45}{0.60} = \$0.75$$

b) P = 40% of \$0.75 = 0.40 x 0.75 = \$0.30. The retail's profit is \$0.30.

4.7 Percent Review Problems

1. Express each of the following as percent.

- a) 10 out of 50 b) 5 out of 25 c) 3 out of 10 d) 260 out of 400
 e) 21 out of 70 f) 18 out of 60 g) 200 of 1000 h) 12 out of 48

2. The value of a one-bedroom condominium is \$80000 and its contents are worth \$3000. Express the value of the contents as a percentage of the value of the house.

3. Don bought a car with sale tax of 14% tax included for \$31,920. In addition, Don bought an extended warranty at 2% of the original purchase price.

- a) What was the price of the car without the sale tax?
 b) How much was the extended warranty?

4. The value of a car decreases as shown in the table below:

Vehicle	Value
New	\$12,000
After 1 year	\$10,000
After 2 years	\$8,800
After 3 years	\$8000

- a) In which year did the value of the car decreased by \$2000?
 b) Calculate the percentage decrease between year 2 and year 3.
 c) During which year was the percentage decrease in the value of the car the greatest?

5. A gas bill of \$46.06 includes GST of 6%. Find the amount of the GST paid.

6. The year-end profit of a company increased this year by 12% to \$90944. What was the profit made last year?

7. After one year, the value of a car has fallen by 15% to \$8330. What was the value of the car at the beginning of the year?

8. A basketball net was originally priced at \$205.00. It has been marked down twice: 10% off the original price, then another 15% off the discounted price.
- What is the new price for the net?
 - What is the single discount rate?
9. Calculate the percentage increase in each of the following cases.
- A transit fare of \$1 is now \$1.20.
 - A taxi fare of \$2.20/km is now 2.35/km.
 - A train fare of \$3.15 is now \$3.50.
10. A clothing store offers \$10 discount to all customers spending \$50 or more. Karen spends \$52.63, and Mike spends \$78.82. Find the percentage saving for Karen and Mike.
11. Ann, an investment broker, bought some shares at a price of \$5.50 each. The price of the shares dropped on the stock market to \$3.50. Find the percentage loss on 200 shares.
12. A retailer lists an item for \$95 less 20% discount. To improve turnover, the item is reduced again to \$60. What additional rate of discount should be offered?
13. Mercy earned \$60,000 in 2001. Her tax allowance was \$6490. She did not pay tax on this amount of her income. On further \$2,500 of her income, she did not pay any income tax because she paid this amount into a pension plan. She, however, paid tax on the rest of her income. The tax rates are as follows: 25% on the first \$30,000 of her taxable income; then 40% on the remaining.
- What was her taxable income? (Note: Taxable income is what remains after all allowances are subtracted.)
 - How much income tax did Mercy pay in 2001?

14. Every year a municipal council determines its budget and decides how much it will tax properties under its area. The process of determining the value of a home is called assessment. A council may use either the historic cost of the property (its cost price) or market cost (How much the property will sell in the market at the time of assessment).

a) Find the assessed value of a store with a market value of \$150,000 if the rate of assessed value is 35% of market value.

b) What is the tax on a property with assessed value of \$188,500 if the tax rate is 4.5% of the assessed value?

c) Jack's farm has a market value of \$400,000. Find the assessed value of the farm if farms are assessed at 25% of the market value.

15. Dora purchased miscellaneous items at Zellers. The item subtotaled \$52.13. The receipt showed \$4.17 in tax paid. What is the tax rate?

16. In an internal audit of 200 sale invoices, the following errors were discovered:

Number of errors:	0	1	2	3	4	5
Number of invoices	60	30	40	40	20	10

What is the percentage of invoices with errors?

17. A new lake is to be stocked with fish according to the numbers in the table below.

Type of fish	A	B	C	D
Number of fish	400	300	200	100

What is the percentage of fish type D would be in the lake?

4.7 Percent Answers

1. a) 20%, b) 20%, c) 30% d) 6.5% e) 30%, f) 30%, g) 20%, h) 25%.

2. 3.75% 3. a) \$27517.24, b) \$550.34. 4. a) Year 1, b) 9.09%, c) Year 1.

5. \$2.61. 6. \$81200 7. \$9800 8. a) 156.83, b) 23.5%. 9. a) 20%, b) 6.82%, c) 11%.

10. Karen 19%; Mike 12.69%. 11. 36.36%. 12. 21.05% 13. a) \$51010, b) \$15904

14. a) \$52500, b) \$ 8482.50, c) \$100000 15. 8 % 16. 70% 17. 10%