Exam II – Fall 2015
Instructor: Lauren Hoehn Velasco

Instructions:

(1) Write answers and number on the provided answer sheet. Include name on LAST page.

(2) Please complete all portions of the exam, and ensure that answers are legible

(3) You must show work to receive credit

(4) Write down proper equations to receive full credit

(5) You may use a piece of paper and calculator for this exam

(6) The exam will end promptly at 8:50/3:50, with the exams removed from all students hands by 8:55/3:55

Question 1: Relationships Between Random Variables (24pts)

Let X and Y be two random variables. Now let Z and W be linear combinations of X and Y such that:

$$W = 4X + 2Y$$

$$Z = 2X - 4Y$$

(a) Find $E(W)$ and $E(Z)$. How are they distinct from one another?

(b) Find $\text{Var}(W)$ and $\text{Var}(Z)$. How are they distinct from one another? What if we knew that X and Y were independent?

(c) Explain in plain english why we care about the statistical properties of linear combinations (giving an example will suffice).

Question 2: Questions from Chapers 4-6 (16pts)

Briefly answer each portion, with knowledge from Chapters 4-6:

(a) What is a probability distribution function? List the distinctions between the continuous case and the discrete case. *Include why in the continuous case we focus on the cdf for calculations.*

(b) What is the difference between a distribution of a sample, and a sampling distribution of a particular statistic?

(c) Write down the equation for (1) the standard error of the mean and (2) the standard error of the sampling distribution of sample proportions. What is the best way to reduce the standard error?

(d) As the size of the sample increases, what happens to the shape of the sampling distribution?

(e) True or false: we need to know the distribution of the population in order for the CLT to apply.

(f) Briefly, in plain English, state the Law of Large numbers (not mathematically).

Question 3: Expected Value and Variance (24pts)

Suppose X denotes the grade point average of college students at a ivy league school, where the pdf is:

$$f(x) = \frac{x}{8},\ 0 < x < 4$$

Assuming that it follows the typical (proper) properties of a pdf, answer the questions a-c.

(a) If $f(x)$ represents a proper pdf, what does $\int_{x_{min}}^{x_{max}} f(x)dx$ equal? Check by evaluating the integral.

(b) Generally, what does $\int_{x_{min}}^{x_0} f(x)dx$ give us? *Do not evaluate!*

(c) Find the expected GPA using integration.

(d) Set up the integral used to find the variance. *Do not evaluate!*

Question 4: Matching Distributions (36pts)

For each of the distributions (a)-(f), answer the following, do not perform any calculations!

1. Which distribution would you use approximate the following real life random variables? **Write** the PDF formula (general).
2. Formally list the key parameters defining shape of distribution (**notation** and **name**)
3. **Sketch** the PDF, include the parameters that shape the distribution. Briefly shows how the parameters listed affect the shape and location of the PDF, include how the distribution would change, if the parameters changed. (*skip change of parameters if exponential*).

(a) The time between arrests in NYC on any given day.

(b) The number of students in a class of 20 who get an A. Where the probability of an A is 0.4.

(c) The amount of time each student at BC spends on homework each weekday, [0,4] hours. Assume equally likely probability across the distribution of hours.

(d) The number of texts you receive per hour, where on average you receive 4.2 texts per hour.

(e) IQ scores for residents of Boston.

(f) Whether or not it rains today.

Bonus Question: Just For Fun

Guess your exam grade (out of 100) and how confident you are (in percentage terms), if you get the precise exam grade you will get a bonus point. Make sure to answer on **the back of the answer sheet**. [1]

[1] The instructor will not look at your guess during grading process, until all questions have been examined so as to not bias grade toward expectation of student.