Lecture # 1

I. LOGIC

1-1. Introduction to discrete Mathematics:

Discrete means made up of distinct part. In mathematics it means discontinuity - Continuous math. deals w/ Diff. Equ. elementary calculus, Geometry ... etc.

Techniques used in both fields are quiet analogus (similar) but the structure of objects we deal with are different - The collection of these objects is called set. Most sets of interest in D.M. are finite or countable.

Discrete and continuous have their counterparts in computer Science can be classified as analog or digital Computers, the difference resides in the counting or measurements of data obtained.

Other examples from real life:

(i) Digital watch ↔ Analog watch.

(ii) Digital signal (Speech, picture) obtained by Sampling and continuous signal.

... etc.

A major reason for the study of D.M. is to acquire the tools and techniques we need to design and understand computer systems.

For further understanding refer to section 1.2.

Reading Assignment: chap. 1 sections 2 & 3.

1-2. Logic and Propositions:

In communication we often use questions, exclamations ... But to communicate facts we use statements. Technically a statement (or proposition) is a sentence that is either true or false.

Exple: consider the following.

Y	(a)	Ten is less than seven
N	(b)	How are you?
N	(c)	she is very talented
Y	(d)	There are life forms on other planets in the universe.
N	(e)	I am a liar

- Connectives and truth values: To vary our conversations, we donot confine ourselves to simple statements. we combine simple statements to make compound statements. The truth value depends on the components values and the connecting words used.

* A common connective is the word "and". symbols for "and" varies in the literature from "$\wedge$" to "&" to "." to " ". The "And" connective is called conjunction.

A and B $\longrightarrow$ conjunction of A and B.

Truth Table for "and"

A	B	$A \cdot B$
T	T	T
T	F	F
F	T	F
F	F	F

truth table for "or"

A	B	$A + B$
T	T	T
T	F	T
F	T	T
F	F	F

* Another connective is the word "or". A or B called disjunction of statements A and B. Symbols also varies "$\vee$", "+", "or".

* A third connective is the "If - then". If A then B symbolized by $A \to B$. It conveys the meaning that the truth of A causes the truth of B.

It may also be expressed as: "A is a sufficient condition for B"
"~~A only if B~~"
" B follows from A "
" B is a necessary condition for A "

A = antecedent.
B = consequent.

Exple: " Fire is a necessary condition for smoke".
can be restated as:

"If there is smoke then there is fire".

Exercice: Rewrite each of the following statement in "if – then" form

(i) A sufficient condition for using a word processor is that the great american Novel is to be written.
(ii) Susan will pass her physics course only if she is bright and studies hard.
(iii) good ~~gasoline~~ combustion is a necessary condition for high gasoline mileage.

The truth table of implication is less obvious; to understand it we use the following example.
Let's suppose you hear your roommate remark:
" If I graduate this fall then I'll take a vacation in Florida "
A = " he graduates this fall "
B = " takes a vacation in Florida "
If A and B are true then the remark was true.
If A is true and B is false the remark was a false statement
Now suppose your Roomate does not graduate. Whether he or she takes a vacation in Florida or not, you could not accuse your roommate of making a false statement. By default we accept $A \to B$ as true if A is false.

Truth table:

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

* The fourth connective is the equivalence ; $A \Leftrightarrow B$, or, $A \leftrightarrow B$
It stands for $(A \to B)$ and $(B \to A)$ which is the "if and only if". A only if B and B only if A.

The truth table :

A	B	$A \to B$	$B \to A$	$A \leftrightarrow B$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

The connectives we have seen so far are called binary connectives. because they join 2 statements.
Now let's consider unary connective, acting only on one statement. The Negation connective.

A is a statement, A' or, not A, means that "it is not true that A", or "It is false that A". It does not mean that A' is always of false value but, that its truth value is opposite to the truth value of A.

Truth table :

A	A'
T	F
F	T

Exples :
(i) let A be : "It will rain tomorrow".
then A' is : "It will not rain tomorrow".
for compound statement it is trickier :
(ii) let B : " Peter is tall and thin "
then B' : " It is not true that Peter is tall and thin "
which may be read :
" Peter is not tall or he is not thin "
that is different from :
" Peter is short and fat ".

(iii) Let C be: " The river is shallow or polluted "

$\Rightarrow$ C' is: " The river is neither shallow nor Polluted."

1.3. Predicate Logic:

In propositional logic we delt only with simple declarative sentences. In general we are interested in more complex and useful propositions that involve quantifying. Quantified statements are called "Predicate logic" or " Predicate calculus". In this System true means Valid, that is, true in all possible interpretations. There are 2 predicate quantifiers:

" for all, $\forall$ " – " There exists, $\exists$ "

* Example ①: Read Exple in the book. (Section 2-2)

② What is the truth value of each of the propositions

(i) $\forall x \; P(x)$

(ii) $\exists x$ such that $P(x)$

Given that $P(x)$ is the predicate " $x = x^2$ "

Ans: $x \neq 1$ and $x \neq 0 \Rightarrow P(x)$ not valid $\Rightarrow$ false.

$x = 1 \Rightarrow P(x)$ is true.

Negation (i) not $\forall x \; P(x) \Leftrightarrow \exists x$ such that not $P(x)$. / (ii) not $\exists x$ s.t. $P(x) \Leftrightarrow \forall x$ not $P(x)$

1.4. Proof techniques:

We have presented so far some basic tools for propositions and predicate, and therefore we are ready to use these tools for constructing logical arguments. Logical arguments are the same thing as proofs. There are many standard ways or methods of proving "things".

① <u>Direct proofs:</u> In general how can we prove that $P \to Q$ is true? The obvious approach is the *direct proof* – Assume the hypothesis P true and deduce the conclusion Q.

<u>Example:</u> " If a number is divisible by 6 then it is divisible by 3" this is about an arbitrary x.

$$(\forall x)(x \text{ divisible by } 6 \longrightarrow x \text{ divisible by } 3)$$

to do this we assume the hypothesis " x is divisible by 6" to be true. Now deduce the second part (Q).
we have to use the definition of divisibility.
"a is divisible by b if $a = q.b$ "

hypothesis : $x = q \cdot 6$ for some k (def. of divisibility)

$6 = 2 \cdot 3$ (number fact)

$x = q.(2.3)$ (substitution)

$x = (q \cdot 2).3$ (property of multiplication)

$(q \cdot 2)$ is an integer (Known from integers properties).

$\Rightarrow x = q' \cdot 3$

$\Rightarrow x$ is divisible by 3.

② <u>Contrapositive proofs:</u> Now suppose we failed to prove our conjecture by direct proof. We may use contraposition based on the fact that " not $Q \Rightarrow$ not P " which is equivalent to $P \to Q$.

<u>Exple:</u> Same statement as in previous example
a proof by contraposition will be :

$Q' \to P'$ " If a number is not divisible by 3 then it is not divisible by 6 ".

hypothesis: "x is not divisible by 3"

$x \neq k \cdot 3$ for any integer k (negation of divisibility by 3)

$x \neq (2k_1)3 \quad \forall k_1$ ($2k_1$ would be an integer k, ruled out about).

$x \neq k_1(2.3) \quad \forall k_1$ (properties of multiplication).

$x \neq k_1 \cdot 6 \quad \forall k_1$ (number fact)

Conclusion: "x is not divisible by 6"

③ Proofs by Contradiction: Another way or technique of proof you may use is "Proof by Contradiction". In this technique you assume both the hypothesis and the negation of the conclusion to be true and then try to deduce some contradiction by using some assumptions.

Exple: Prove by contradiction:

"If a number is added to itself gives itself then the number is zero".

Let x represent any number:

hypothesis: $x + x = x$

conclusion: $x = 0$.

by contradiction; assume $x + x = x$ and $x \neq 0$

then $2x = x$ and $x \neq 0$

Because $x \neq 0$ divide both sides by x.

$\Rightarrow \quad 2 = 1 \quad \Rightarrow$ Contradiction

Hence $x + x = x \quad \Rightarrow \quad x = 0$

④ Proof by Counterexamples: This technique is helpful if we want to prove non validity of the predicate $\forall x\, P(x)$. Find at least one value of x which disapproves $P(x)$.

Exple: $\forall x\ P(x)$, $P(x)$: $x = \sqrt{x}$ $\quad x = 2$. $P(x)$ not true.

1.5 Mathematical Induction:

- Principle of Mathematical Induction;

PMI is useful for proving propositions that must be true for all integers or for a range of integers. The general form of the problem situation to which we apply PMI is the following:

"Prove $P(n)$ for all $n \geq k$."

or

"For all positive integers shat that ....."

General statement of PMI is:

Let $P(n)$ be a proposition that is valid for $n \geq k$
n, k integers.

if: ① $P(k)$ is true and (Base step)

② $\forall n \geq k$, $P(n) \Longrightarrow P(n+1)$ (Induction step)

then $P(n)$ is true $\forall n \geq k$.

<u>Exple ①</u>: Prove that the equation

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

is valid for any integer n.

① $P(1)$ Base : $P(1) = 1 = 1^2 = 1$. true.

② Assume $P(k)$ is true :

$$P(k) = 1+3+5+\cdots+(2k-1) = k^2$$

and try to show $P(k+1) = 1+3+5+\cdots+[2(k+1)-1] \overset{?}{=} (k+1)^2$

$$P(k+1) = 1+3+5+\cdots+(2k-1)+(2k+1)$$
$$= k^2 + 2k+1 = (k+1)^2$$

Exple 2 Prove that :

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1 \qquad \forall n \geq 1$$

Again Induction is appropriate :

-1- Base $P(1)$:

$$1 + 2 = 3 = 2^2 - 1 = 3 \quad \text{true}$$

-2- Take $P(k)$ (or Assume $P(k)$) and try to establish $P(k+1)$

$$1 + 2 + \cdots + 2^k = 2^{k+1} - 1$$

and try to establish $P(k+1)$: $1 + 2 + \cdots + 2^{k+1} = 2^{k+1+1} - 1$

$$1 + 2 + \cdots + 2^k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$$

$$= 2 \cdot 2^{k+1} - 1 = 2^{k+1+1} - 1$$

Q.E.D

Reading Assignment : Read "Inductive Definitions"

HW. Assgnt

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