Chapter II

SETS

Read from 2.6 / 2-8

2.1. Basic Properties of Sets:

Set theory is one of the cornerstones of Mathematics. Many concepts in Math. & C.S^ce^. can be conveniently expressed in the language of sets. Operation can be performed to generate new sets.

2.1.1. Definitions of Sets:

A set is a collection of objects, which are called the elements of the set. These objects are characterized by some defining property: Thus any given object either does or does not have the property and, Thus, either does or does not belong to the set.

2.1.2. Notation:

Capital letters denote sets

$\in$: denotes membership in a set

Braces: $\{\ \}$ are used to ~~denote~~ indicate a set.

exple: $A = \{\text{Red, Blue, yellow}\}$, Red $\in A$
green $\notin A$.

2.1.3. Examples and Special Sets:

$A = \{4, 5, 6, 7\} = \{x \mid x \text{ is an integer and } 3 < x \leq 7\}$

$B = \{\text{Jan, Mar, May, Jul, Aug, Oct, Dec}\} = \{x \mid x \text{ is a month w/ exactly 31 days}\}$

$C = \{x \mid x = y^3, \text{ for } y \in \{0, 1, 2\}\} = \{0, 1, 8\}$.

- $\mathbb{N}$ = The set of all nonnegative integers = $\{\ 1, 2, 3, \ldots\}$
- $\mathbb{Z}$ = " " " " integers = $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$
- $\mathbb{Q}$ = " " " " rational numbers = $\{\frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \ldots\} = \{\frac{n}{m} \mid n \in \mathbb{Z}, m \in \mathbb{Z}^*\}$
- $\mathbb{R}$ = " " " " real #'s.

- ϕ : the empty set, or null set. $\phi = \{\ \}$

2.1.4. Relationships Between sets:

- Equality: 2 sets are said to be equal if they contain the same elements.

Exple: $A = \{1, 2, 3\}$
$B = \{2, 1, 3\}$ $\quad A = B = C$.
$C = \{3, 2, 1\}$

- Subset: Let S be a set, A subset of S is a set A such that if $x \in A$ then $x \in S$. We write it.

$$A \subseteq S \qquad x \in A \Rightarrow x \in S$$

Exple:

$A = \{1, 7, 9, 15\}$
$B = \{7, 9\}$
$C = \{7, 9, 15, 20\}$

The following statement are all true:

$B \subseteq C$, $B \subseteq A$, $B \subset A$, $A \not\subseteq C$

$15 \in C$, $\{7, 9\} \subseteq B$, $\{7\} \subset A$, $\phi \subseteq C$

$(x \in \phi \Rightarrow x \in C)$
(false $\rightarrow$?) True.

- Operations on sets:

① Union: $A \cup B = \{x;\ x \in A \text{ or } x \in B\}$

Exple: $A = \{1, 2\}$
$B = \{2, 3\}$ $\quad A \cup B = \{1, 2, 3\}$.

② Intersection:

$$A \cap B = \{x;\ x \in A \text{ and } x \in B\}$$
$$= \{2\}.$$

③ Complement: Complement of set A relative to set B is the set

$$B - A = \{x\ ;\ x \in B \text{ and } x \notin A\}.$$

④ Venn diagram: Picture that illustrates set operation in universal set.

- <u>Properties and identities:</u> Let $\mathcal{U}$ denote the universal set containing A, B, C.

Identity is an equality that is true for all sets.

<u>Theorem</u>: 2-1

1. $\phi \subseteq A \quad \forall A.$
2. $A \subseteq A \quad \forall A.$
3. If $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$
4. $\text{not}(\text{not } A) = \sim(\sim A) = \overline{\overline{A}} = A \quad \forall A$ (Involution property)
5. $A \cup \sim A = A \cup \overline{A} = \mathcal{U}$, & $A \cap \overline{A} = \phi \quad \forall A$
6. $A \cup \mathcal{U} = \mathcal{U}$, &, $A \cap \mathcal{U} = A \quad \forall A$
7. $A \cup \phi = A$ and $A \cap \phi = \phi \quad \forall A$
8. De Morgan's Laws: For all sets A and B.

$$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$$

and

$$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

<u>Set Identities:</u>

<u>Theorem</u>: 2.2 - Let A, B, C be sets.

1. a) $A \cup A = A$ b) $A \cap A = A$
2. a) $A \cup (B \cup C) = (A \cup B) \cup C$ b) $A \cap (B \cap C) = (A \cap B) \cap C$.
3. a) $A \cup B = B \cup A$ b) $A \cap B = B \cap A$
4. a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

<u>Duality Principle:</u>

<u>Theorem</u>: 2-3 Any Equality involving sets can be changed to an equally valid <u>dual</u> equality, by applying the following rules:

- Replace each $\cup$ by $\cap$ and each $\cap$ by $\cup$
- " " $\mathcal{U}$ by ϕ " " ϕ " $\mathcal{U}$

2.1.5 : More on sets: The number of elements in a set S is called "cardinality of set S" denoted $|S|$ can be finite or infinite.

exple : $S_n = \{1, 2, \dots, n\}$ is finite.

$\mathbb{N}, \mathbb{Q}, \mathbb{Z}, \mathbb{R}$, are infinite.

Theorem 2.4: Let A & B be finite sets:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

• Power Sets : For a set S we can form a new set whose elements are all of the subsets of S. The new set is called The power set of S. Denoted by $\mathcal{P}(S)$

exple: $S = \{1, 2\} \Rightarrow \mathcal{P}(S) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$.

$S = \{a\} \Rightarrow \mathcal{P}(S) = \{\phi, \{a\}\} = \{\phi, S\}$.

Theorem 2.5: If S is a set with $|S| = n$, n: finite. then $|\mathcal{P}(S)| = 2^n$ (Proof by Induction).

• Product Set:

Before we get into product set, we must define ordered pair. An ordered pair is a pair of objects distinguishable by their order (a, b). Two ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.

•• Cartesian Product : The cartesian product of A & B denoted $A \times B$, is the set of all ordered pairs (a, b) such that $a \in A$, and $b \in B$.

$$A \times B = \{(a, b) \mid a \in A, b \in B\}.$$

Exple: $A = \{1, 2\}$, $B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$B \times A = \{(3,1), (3,2), (4,1), (4,2)\}$

$A \times A = A^2 = \{(1,1), (1,2), (2,1), (2,2)\}$.

$A \times A \times A = A^3 = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$

<u>Theorem 2.6</u>: Let A & B be sets:

if $|A| = n$,
$|B| = m$, then $|A \times B| = n \cdot m$

- <u>Partitions of a Set</u>:

Let S be a set; A partition of S is a set:

$$\pi = \{A_1, A_2, \dots, A_i, \dots, A_k\}$$

where:

1. each A_i is a non empty subset of S, $i = 1, 2, \dots k$.
2. The A_i cover S:
   $S = A_1 \cup A_2 \cup \dots \cup A_k$.
3. The A_i's are mutually disjoint in that if $i \neq j$ then $A_i \cap A_j = \phi$

A_i : is called block i of a partition.

<u>Exple</u>: $S = \{a, b, c, d, e, f, g, h\}$.

Let $A_1 = \{a\}$. $A_3 = \{c, f, h\}$.
$A_2 = \{b, g\}$ $A_4 = \{d, e\}$
are blocks of a partition.

<u>HW. Assgn #2.</u>

Reading 2.6 2.7

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