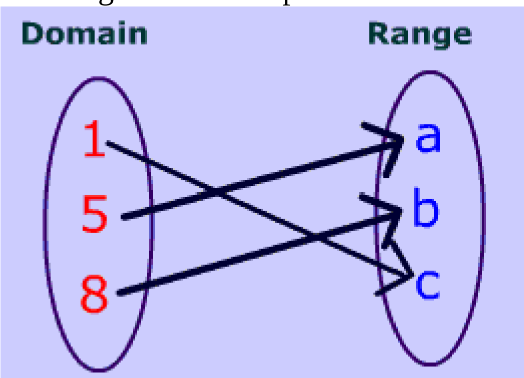
Question 1.

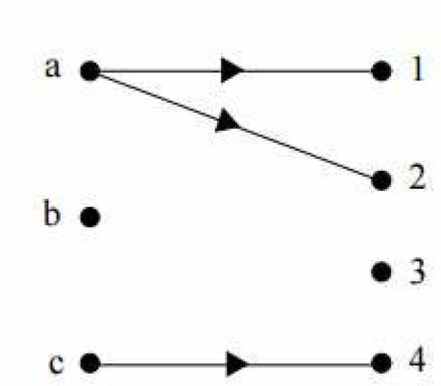
The diagram below represents relation R:



R is defined by: [answer box]

answers: right one -> R={{1,c},{5,a},{8,b}}
wrong -> R = {{c,1},{a,5},{b,8}}
wrong -> R = {1,5,8,a,b,c}

<u>Variation :</u>



answers: right one -> R={{a,1},{a,2},{c,4}}
wrong -> R={{1,a},{2,a},{4,c}}
wrong -> R={a,b,c,1,2,3,4}

Question 2 :definitions

Let A be a set and let R be a binary relation on A. R is **reflexive** if [answer box]
answers: Right answer -> $\forall x[(x \in A) \rightarrow ((x,x) \in R)]$.

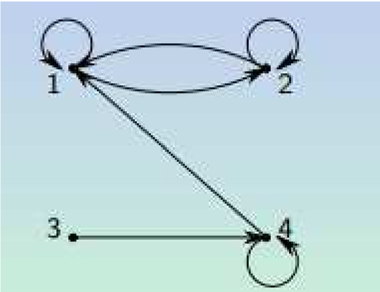
Variations:
1.
R *is* **symmetric** *if*
$\forall x \forall y[((x,y) \in R) \rightarrow ((y,x) \in R)]$.

2.

R *is* **transitive** *if*
$\forall x \forall y \forall z[([(x,y) \in R] \wedge [(y,z) \in R]) \rightarrow ((x,z) \in R)]$.

Question 3:

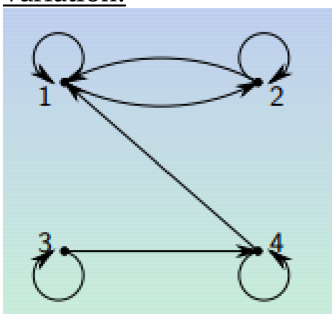
a) If this relation R is reflexive?



answer: [answer box] : yes or no

right answer : NO

<u>variation:</u>

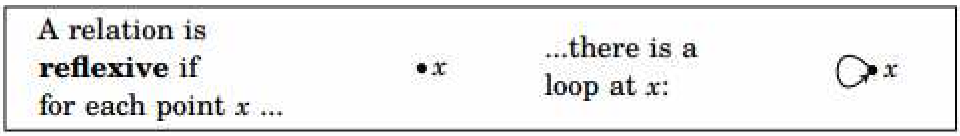


answer : [answer box]: yes or no

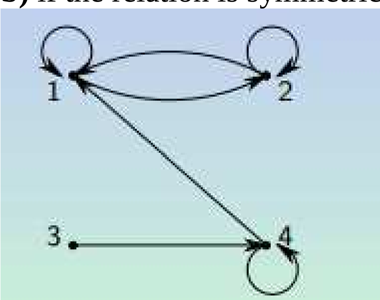
right answer: YES

(YES answer will be always where all vertices have self loops)

1.



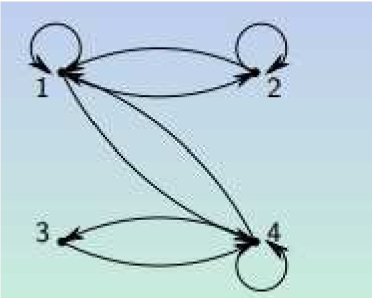
b) if the relation is symmetric:



answer [answer box}: yes or no

right answer: NO

<u>variation:</u>



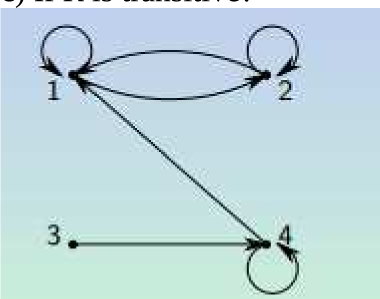
answer [answer box]: yes or no

right answer: YES

(YES answer, there must be an edge in both directions [, if there is an edge from 1 to 2, there should be also from 2 to 1])

2. A relation is **symmetric** if whenever there is an arrow from x to y ... $x \to y$ ...there is also an arrow from y back to x: $x \leftrightarrows y$

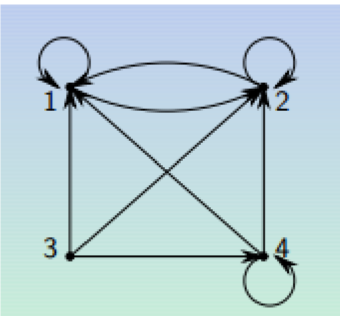
c) If R is transitive?



answer [answer box}: yes or no

right answer: NO

variation:

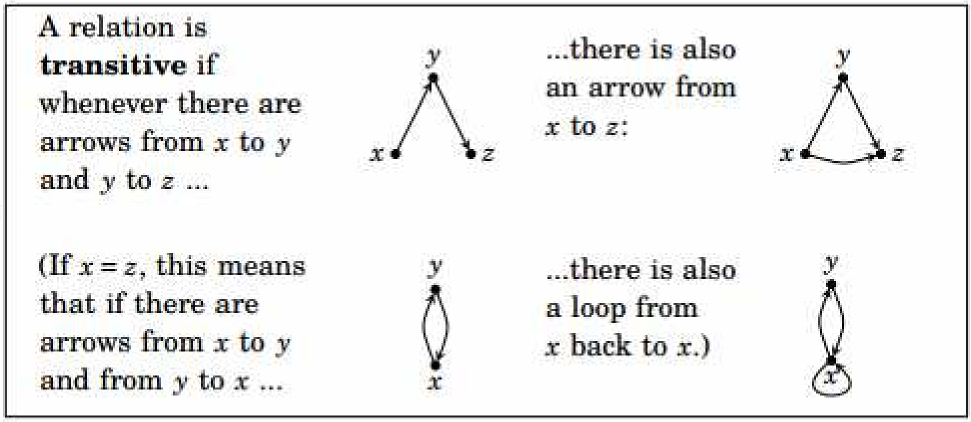


answer : [answer box]: yes or no

right answer: YES

(YES answer when "triangular paths" must be closed.)

3.



Question 4.

If M below is a matrix representation of Relation R,

M= 1 1 0
1 0 0
0 1 0

R is defined by: [answer box]
right answer -> R={(1,1),(1,2),(2,1),(3,2)}
R={(1,3),(2,2),(2,3),(3,1),(3,3)}
R={2,1,1}

Question 5

Let A={1,2,3}, B={1,2,3,4},
R1={(1,1),(2,2),(3,3)} and R2={(1,1),(1,2),(1,3),(1,4)}
then

R1 and R2 = {(1,1),(1,2),(1,3),(1,4),(2,2),(3,3)}
R1 or R2 = {(1,1)}
R1 - R2 ={(2,2),(3,3)}
R2 - R1 = {(1,2),(1,3),(1,4)}

Variations: change the set A, B, R1 or R2.

Question 6

EXAMPLE 1.6.2. *Suppose T is the relation on the set of integers given by xTy if $2x - y = 1$. This relation is...*

- *not reflexive*
- *not irreflexive*
- *not symmetric*
- *antisymmetric*
- *not asymmetric*
- *not transitive*

Variation:

EXAMPLE 1.6.3. *Suppose* $A = \{a, b, c, d\}$ *and* R *is the relation* $\{(a, a)\}$. *This relation is...*

- *not reflexive*
- *not irreflexive*
- *symmetric*
- *antisymmetric*
- *not asymmetric*
- *transitive*

EXAMPLE 1.7.2. *Recall Example 1.2.2:* $R_2 \subset \mathbb{N} \times \mathbb{N}$ *was defined by* $(m, n) \in R_2$ *if and only if* $m|n$.

- *reflexive*
- *not irreflexive*
- *not symmetric*
- *antisymmetric*
- *not asymmetric*
- *transitive*

Question 7

Find inverse relation of R (R with dash) and complement relation of R (R ^{-1}) , where R is defined by :

Example

Let $A = \{1, 2, 3, 4, 5\}$ and $R : A \leftrightarrow A :\equiv \{(a, b) : a \mid b\}$. What are $\overline{R}$ and R^{-1}?

Solution

- $R = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,4), \\ (3,3), (4,4), (5,5) \end{array} \right\}$
- $\overline{R} = \left\{ \begin{array}{l} (2,1), (2,3), (2,5), (3,1), (3,2), (3,4), (3,5), \\ (4,1), (4,2), (4,3), (4,5), (5,1), (5,2), (5,3), \\ (5,4) \end{array} \right\}$
- $R^{-1} = \left\{ \begin{array}{l} (1,1), (2,1), (3,1), (4,1), (5,1), (2,2), (4,2), \\ (3,3), (4,4), (5,5) \end{array} \right\}$

Question 8

Full fill the tables using yes or no :

Relations on $\mathbb{Z}$:	$<$	$\leq$	$=$	$\mid$	$\nmid$	$\neq$
Reflexive	no	yes	yes	yes	no	no
Symmetric	no	no	yes	no	no	yes
Transitive	yes	yes	yes	yes	no	no

variations: (changing order of column)