Question 1. Samples

Example: in the lock above, there are 10 numbers to choose from (0,1,...9) and we choose 3 of them:

$$10 \times 10 \times \ldots \text{(3 times)} = 10^3 = 1.000 \text{ permutations}$$

variation

A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?

A 492,804	B 650,000
C 676,000	D 1,757,600

Excellent ... you are right. +0.50

The number of ways of choosing the letters = 26 × 26 = 676
The number of ways of choosing the digits = 10 × 10 × 10 = 1,000

So the number of possible passwords = 676 × 1,000 = 676,000

Question 2. Permutation

For example, what order could 16 pool balls be in?

After choosing, say, number "14" we can't choose it again.

So, our first choice has 16 possibilites, and our next choice has 15 possibilities, then 14, 13, etc. And the total permutations are:

$$16 \times 15 \times 14 \times 13 \times \ldots = 20.922.789.888.000$$

But maybe we don't want to choose them all, just 3 of them, so that is only:

$$16 \times 15 \times 14 = 3.360$$

variation:

How many ways can first and second place be awarded to 10 people?

$$\frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{3.628.800}{40.320} = 90$$

(which is just the same as: **10 × 9 = 90**)

variation

A special type of password consists of four **different** letters of the alphabet, where each letter is used only once. How many different possible passwords are there?

A 4^{26}

B 456,976

C 14,950

D 358,800

You got it Right! +0.50

The number of permutations of 4 letters chosen from 26 is ${}^{26}P_4 = 26 \times 25 \times 24 \times 23 = 358{,}800$

Question 3. Combinatios

Example The number of possible combinations of 3 objects from 5 is

$$C_{5,3} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)} = \frac{5 \cdot 4}{2 \cdot 1} = 10$$

variation:

How many different committees of 5 people can be chosen from 10 people?

A 252

B 2,002

C 30,240

D 100,000

Yes! That is Right! +0.50

In choosing a committee, order doesn't matter; so we need the number of combinations of 5 people chosen from 10

$= {}^{10}C_5$

$= 10!/(5!)(5!)$

$= (10 \times 9 \times 8 \times 7 \times 6)/(5 \times 4 \times 3 \times 2 \times 1)$

$= 30{,}240/120$

$= 252$

Question 4 Selections

Example The number of possible combinations with repetition of 3 objects from 5 is

$$
\begin{aligned}
C'_{5,3} &= \frac{(5+3-1)!}{(5-1)!3!} \\
&= \frac{7!}{4!3!} \\
&= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} \\
&= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \\
&= 7 \cdot 5 = 35
\end{aligned}
$$

Question 5. Numbers of words in VOLVO is:

5!/(2! 2!) = 30

variations: changing words VOLVO, ANNA, etc

We could add examples from lecture with changed context as a variations of questions also.