Properties of Logarithms

$a > 0 \quad a \neq 1$

$\log_a 1 = 0$ because $a^0 = 1$

$\log_a a = 1$ because $a^1 = a$

Logarithms & Exponentials are inverse functions of each other.

$$a^{\log_a M} = M \quad (1)$$

$$\log_a a^r = r \quad (2)$$

(3) $\log_a(MN) = \log_a M + \log_a N$

(4) $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

(5) $\log_a M^r = r\log_a M$

(6) $a^r = e^{r\ln a} = e^{\ln a^r} = a^r$

(14) $\log_2 2^{-13} = -13$

(16) $\ln e^{\sqrt{2}} = \sqrt{2}$

(20) $\log_6 9 + \log_6 4 = \log_6 (9 \cdot 4)$

$= \log_6 36 = 2$

$= \log_6 6^2 = 2$

(22) $\log_8 16 - \log_8 2 = \log_8 \left(\frac{16}{2}\right) = \log_8 8 = 1$

$\ln 2 = a \qquad \ln 3 = b$

(30) $\ln \frac{2}{3} = \ln 2 - \ln 3$

$= a - b$

(32) $\ln 0.5 = \ln \frac{1}{2} = \ln 1 - \ln 2$

$= 0 - a = -a$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a M^r = r\log_a M$$

(48) $\ln\left(x\sqrt{1+x^2}\right) = \quad x > 0$

$\ln x + \ln(1+x^2)^{1/2} =$

$\ln x + \frac{1}{2}\ln(1+x^2)$

(42) $\ln\frac{e}{x} = \ln e - \ln x = 1 - \ln x$

(52) $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right] = \quad x > 2$

$\log\left(x^3\sqrt{x+1}\right) - \log(x-2)^2 =$

$\log x^3 + \log(x+1)^{1/2} - 2\log(x-2) =$

$3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$

(60)

$$\log_2\left(\frac{1}{x}\right)+\log_2\left(\frac{1}{x^2}\right)=\log_2\left(\frac{1}{x}\cdot\frac{1}{x^2}\right)$$

$$=\log_2\left(\frac{1}{x^3}\right)$$

(64)

$$\log\left(\frac{x^2+2x-3}{x^2-4}\right)-\log\left(\frac{x^2+7x+6}{x+2}\right)=$$

$$\log\left(\frac{x^2+2x-3}{x^2-4}\div\frac{x^2+7x+6}{x+2}\right)=$$

$$\log\left(\frac{\overset{(x+3)(x-1)}{\cancel{x^2+2x-3}}}{\cancel{(x+2)}(x-2)}\cdot\frac{\cancel{(x+2)}}{\underset{(x+6)(x+1)}{\cancel{x^2+7x+6}}}\right)=$$

$$\log\left(\frac{(x+3)(x-1)}{(x-2)(x+6)(x+1)}\right)$$

$\ln e = 1$ $\log_4 16 = 2$

If $a^u = a^v$, then $u = v$.

If $\log_a M = \log_a N$, then $M = N$

If $M = N$, then $\log_a M = \log_a N$

Change of Base Formula

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

72 $\log_5 18 = \frac{\log 18}{\log 5} = \frac{\ln 18}{\ln 5}$

$1.796 = 1.796$

78 $\log_\pi \sqrt{2} = \frac{\log \sqrt{2}}{\log \pi} = \frac{\ln \sqrt{2}}{\ln \pi}$

$= 0.303 \quad = 0.303$

If $\log_a M = \log_a N$, then $M = N$

(88) $\ln y = \ln(x+C)$

$y = x + C$

(90) $\ln y = 2\ln x - \ln(x+1) + \ln C$

$\ln y = \ln x^2 - \ln(x+1) + \ln C$

$\ln y = \ln\left(\dfrac{x^2}{x+1}\right) + \ln C$

$\ln y = \ln\left(\dfrac{Cx^2}{x+1}\right)$

$y = \dfrac{Cx^2}{x+1}$